

# Διόρθωση στην Εκτίμηση Μέγιστης Ετήσιας Βροχόπτωσης

Δρ. Γιάννης Διαλυνάς  
Λέκτορας  
Τμήμα Πολιτικών Μηχ. & Μηχ. Περιβάλλοντος  
Πανεπιστήμιο Κύπρου

# EXTREME HYDROLOGIC EVENTS



Mandra, Greece, Nov. 2017

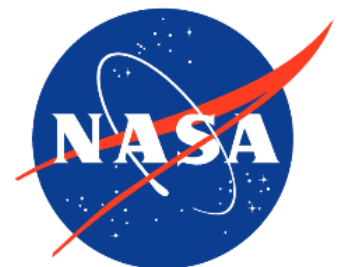


Nicosia, Cyprus, Dec. 2014

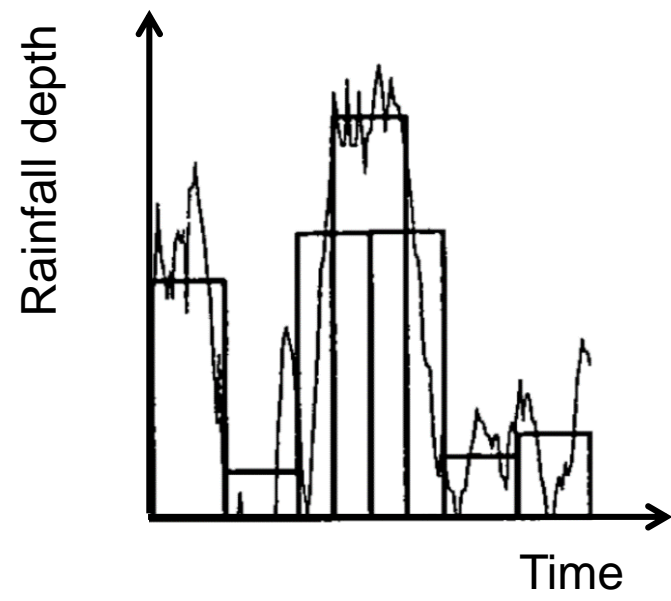


Mandra, Greece, June 2018

# MEASURING PRECIPITATION DEPTH

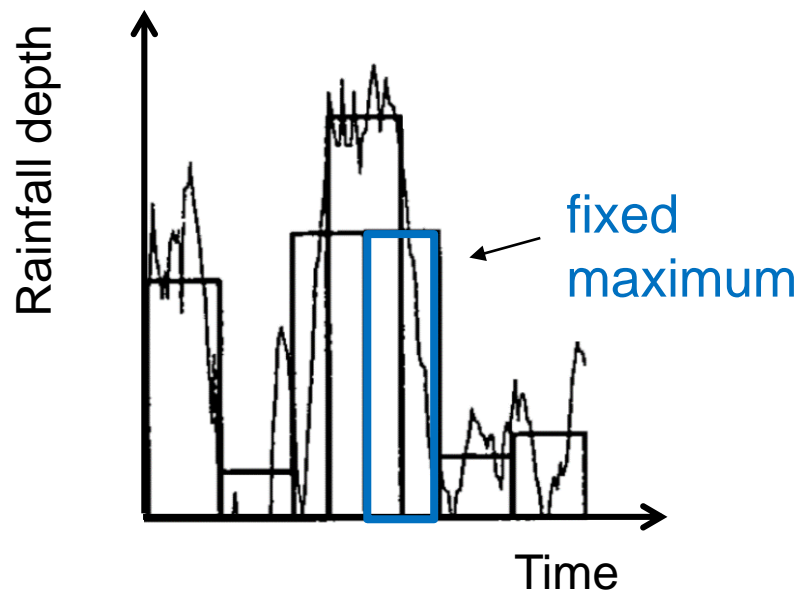


# FIXED AND SLIDING MAXIMA

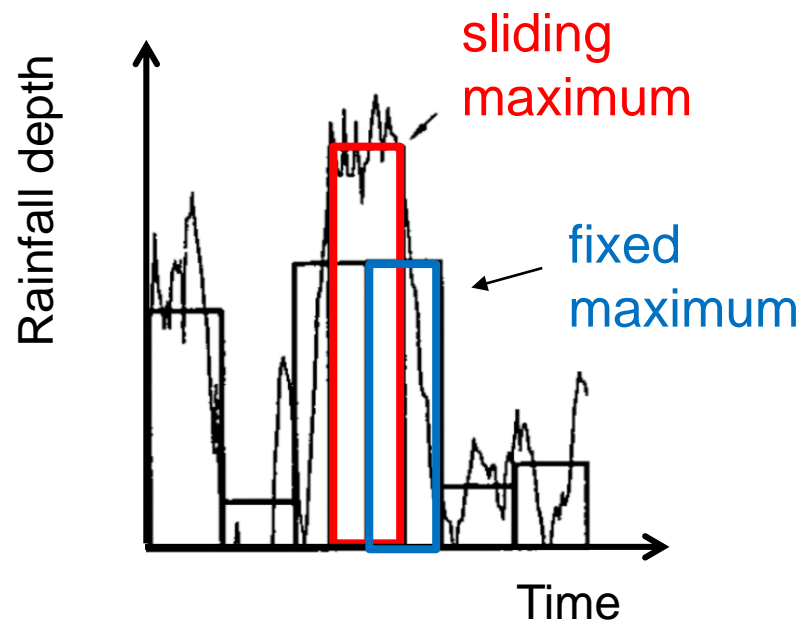


# FIXED AND SLIDING MAXIMA

$$F := \max \left\{ \int_t^{t+1} I(\tau) d\tau \right\}_{t=0,1,\dots,T-1} \quad \text{Fixed Maximum}$$



# FIXED AND SLIDING MAXIMA



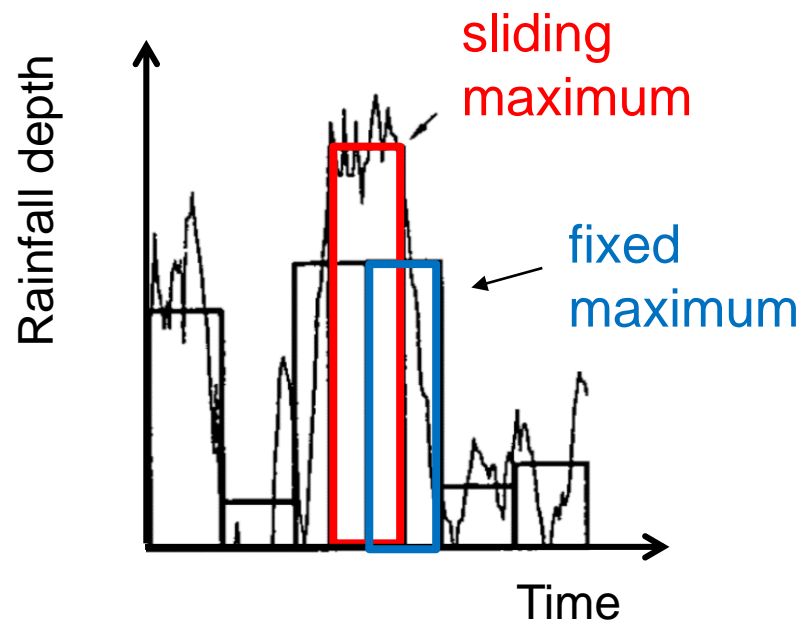
$$F := \max \left\{ \int_t^{t+1} I(\tau) d\tau \right\}_{t=0,1,\dots,T-1}$$

Fixed Maximum

$$S := \max \left\{ \int_t^{t+1} I(\tau) d\tau \right\}_{0 < t < T-1}$$

Sliding Maximum

# FIXED AND SLIDING MAXIMA



$$F := \max \left\{ \int_t^{t+1} I(\tau) d\tau \right\}_{t=0,1,\dots,T-1} \quad \text{Fixed Maximum}$$

$$S := \max \left\{ \int_t^{t+1} I(\tau) d\tau \right\}_{0 < t < T-1} \quad \text{Sliding Maximum}$$

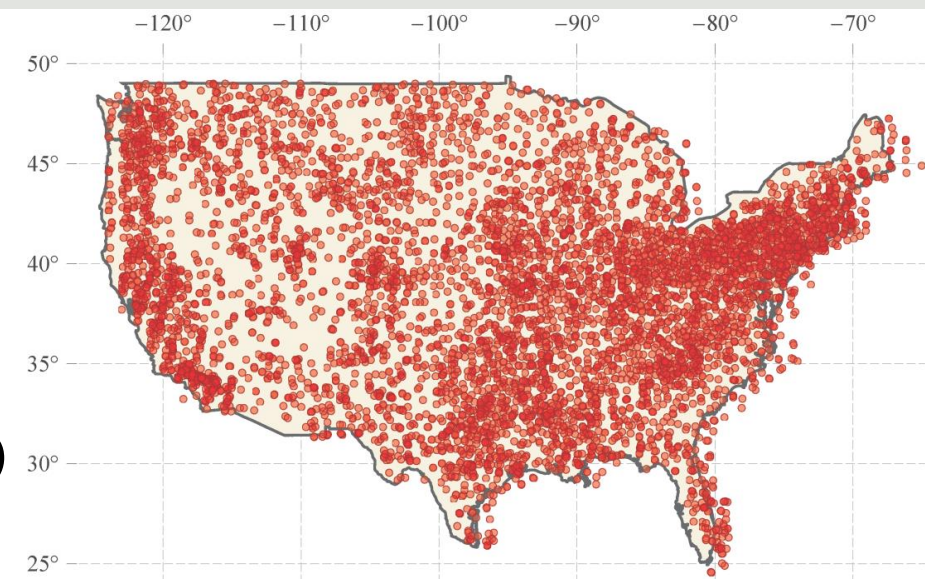
$$H_n = \frac{S_n}{F_n} \quad \text{Hershfield's Correction Factor}$$

# DATASET

- We analyzed a very large database from the National Climatic Data Center (NCDC) that comprises **hourly** precipitation data from thousands of stations from all over the United States.
- The records are of variable length ranging from just a few years to more than 50 years.
- All data used at larger than the hourly time scale were constructed by aggregating the original hourly time series.
- We studied **7.127 records** of hourly precipitation and we estimated the sliding and fixed-interval maximum precipitation for every year and for every record. The result was an unprecedentedly large number of estimated values, i.e., more than 100.000 years for each time scale.

- **Previous studies are based on *limited datasets***

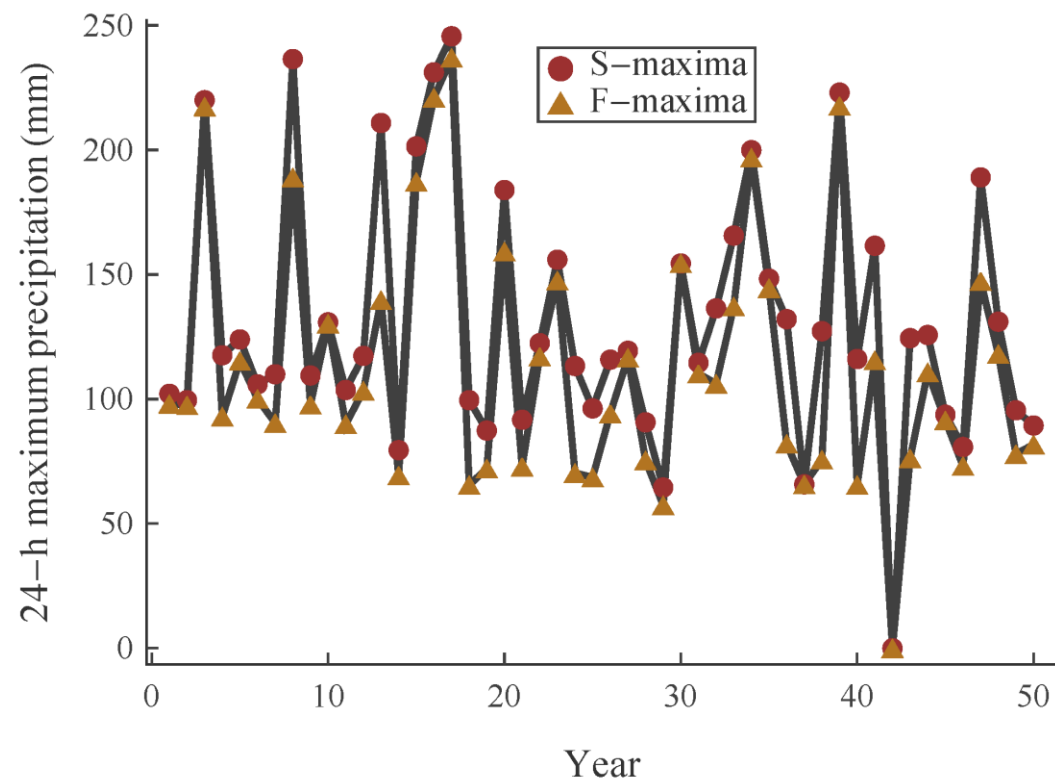
- Kerr et al. (1970): **45** stations (PA, USA)
- Harihara and Tripathi (1973): **67** stations, 25 years (India)
- Natural Environment Research Council (1975): **50** stations (UK)
- van Montfort (1990): **1** station, 58 years
- Huff and Angel (1992): **41** stations, 40 years (IN, USA)
- Faiers et al. (1994): **14** stations (LA, USA)



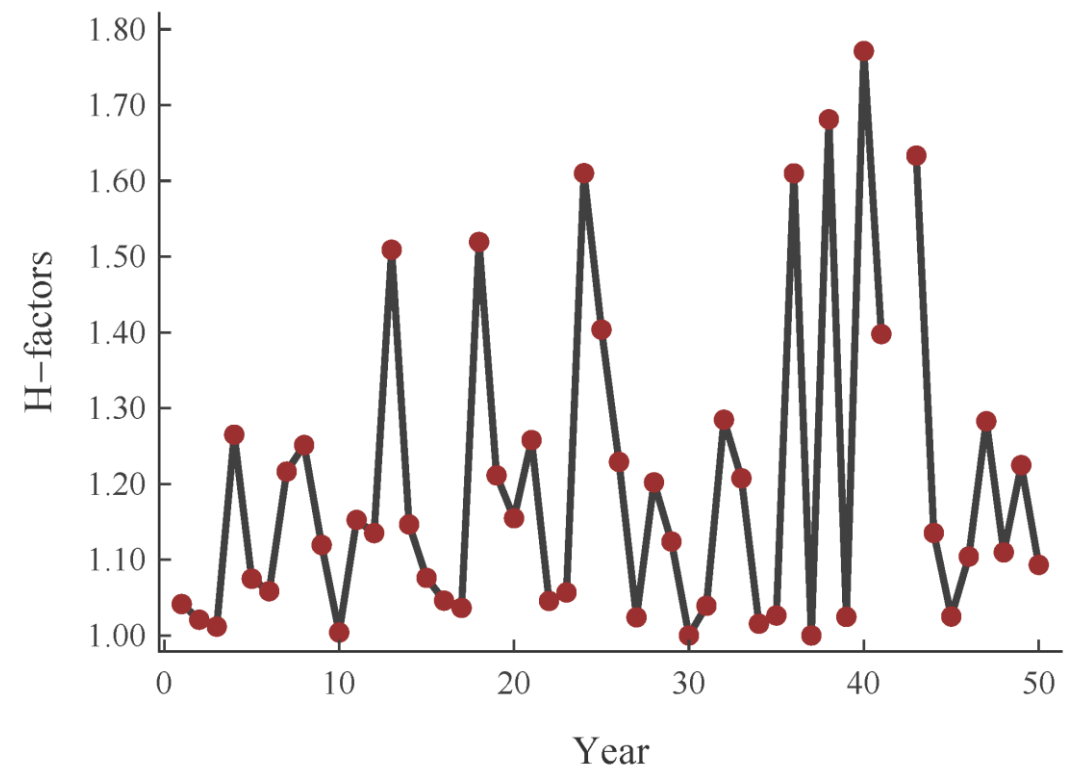


# METHODOLOGY

- For all records we estimated:
  - i. The sliding maximum value at several time scales  $k$  by sliding a  $k$ -hour moving window over the year and extracting the maximum of the resulted values.
  - ii. The fixed-interval maximum value by aggregating the hourly values over the year in  $k$ -hour non-overlapping intervals and extracting the maximum of the resulted values.
- We estimated the  $H$ -factors as the ratio of the S-maximum value to the F-maximum value.
- We performed this analysis for the following time scales given in hours: {2, 4, 6, 8, 12, 16, 20, 24, 30, 36, 42, 48, 56, 64, 72}.



Sliding and fixed interval maxima of a randomly selected time series.

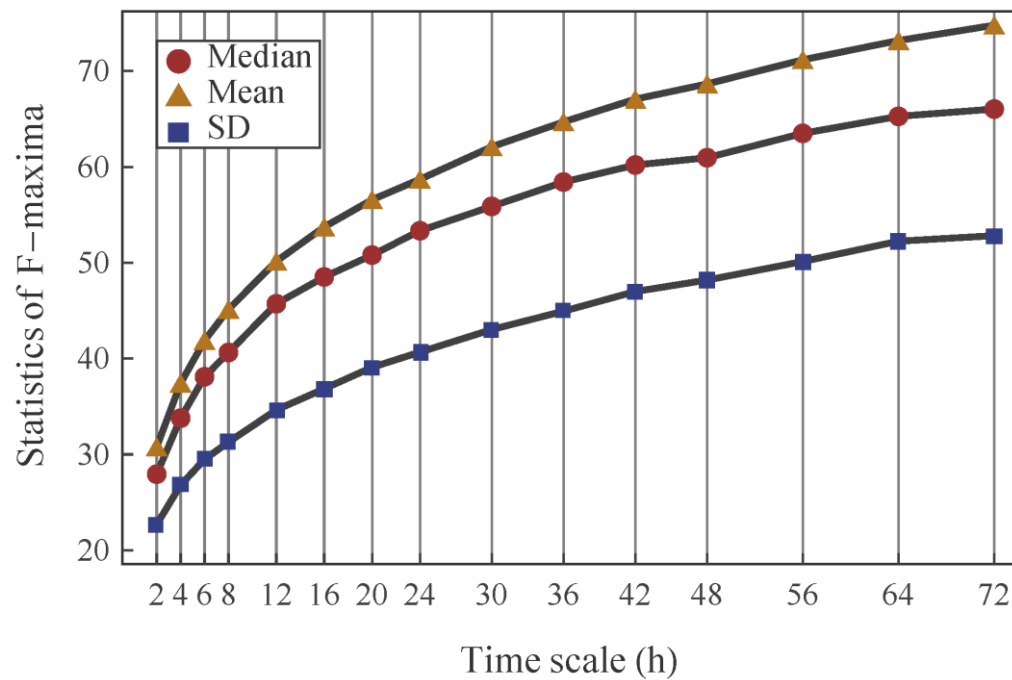


Estimated 24 h  $H$ -factors from Fig. 3 as the ratio of S- to F-maxima.

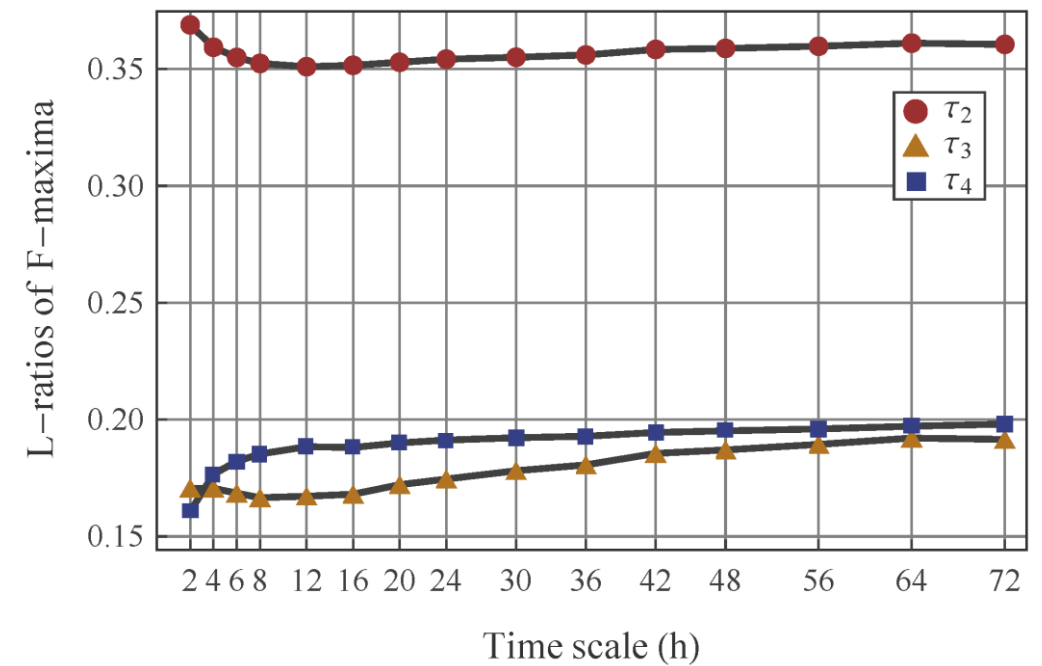
# STATISTICS OF F-MAXIMA

**Table 1.** Basic summary statistics of fixed-interval maxima at various time scales

Scale	No of yrs	Median	Mean	SD	$C_V$	$C_s$	$C_k$	$\tau_2$	$\tau_3$	$\tau_4$
2-hr	129634	27.94	30.78	22.69	0.74	4.07	76.45	0.37	0.17	0.16
4-hr	129634	33.78	37.43	26.80	0.72	4.21	105.20	0.36	0.17	0.18
6-hr	129634	38.10	41.88	29.48	0.70	4.41	153.11	0.35	0.17	0.18
8-hr	129634	40.64	45.14	31.28	0.69	3.16	59.60	0.35	0.17	0.19
12-hr	129634	45.72	50.15	34.64	0.69	3.47	88.85	0.35	0.17	0.19
16-hr	129634	48.51	53.73	36.85	0.69	2.57	36.10	0.35	0.17	0.19
20-hr	129634	50.80	56.58	39.09	0.69	2.93	58.96	0.35	0.17	0.19
24-hr	129634	53.34	58.71	40.67	0.69	2.78	51.20	0.35	0.17	0.19
30-hr	129613	55.88	62.10	43.03	0.69	2.60	42.69	0.36	0.18	0.19
36-hr	129613	58.42	64.70	44.94	0.69	2.50	37.37	0.36	0.18	0.19
42-hr	129613	60.20	67.07	47.00	0.70	2.46	33.47	0.36	0.19	0.19
48-hr	129613	60.96	68.66	48.19	0.70	2.43	31.45	0.36	0.19	0.20
56-hr	129610	63.50	71.18	50.13	0.70	2.39	28.92	0.36	0.19	0.20
64-hr	129610	65.28	73.19	52.25	0.71	3.85	134.88	0.36	0.19	0.20
72-hr	129610	66.04	74.78	52.82	0.71	2.32	25.31	0.36	0.19	0.20



Summary statistics of sliding maxima at various time scales.

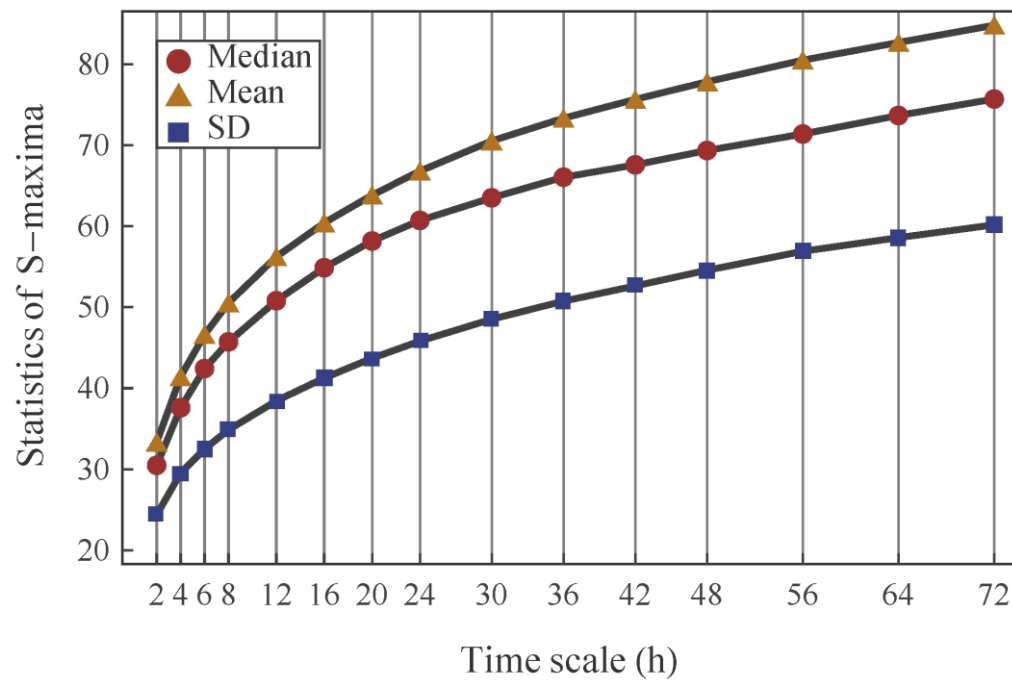


L-ratios  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  of sliding maxima at various time scales.

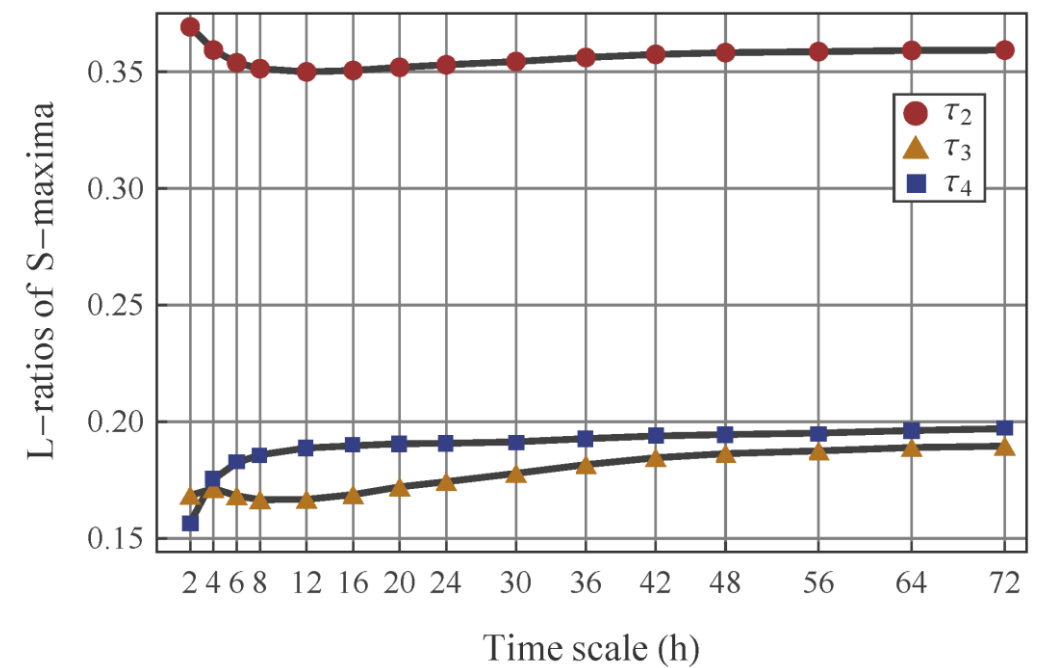
# STATISTICS OF S-MAXIMA

**Table 2.** Basic summary statistics of sliding maxima at various time scales

Scale	No of yrs	Median	Mean	SD	$C_V$	$C_s$	$C_k$	$\tau_2$	$\tau_3$	$\tau_4$
2-hr	129634	30.48	33.33	24.36	0.73	3.86	74.39	0.37	0.17	0.16
4-hr	129634	37.59	41.43	29.52	0.71	4.64	161.36	0.36	0.17	0.18
6-hr	129634	42.42	46.61	32.50	0.70	3.87	112.92	0.35	0.17	0.18
8-hr	129634	45.72	50.52	34.86	0.69	3.46	88.00	0.35	0.17	0.19
12-hr	129634	50.80	56.22	38.44	0.68	2.98	62.64	0.35	0.17	0.19
16-hr	129634	54.86	60.41	41.25	0.68	2.71	49.35	0.35	0.17	0.19
20-hr	129634	58.17	63.83	43.70	0.68	2.55	41.23	0.35	0.17	0.19
24-hr	129634	60.71	66.80	45.81	0.69	2.42	35.50	0.35	0.17	0.19
30-hr	129613	63.50	70.52	48.51	0.69	2.30	30.09	0.35	0.18	0.19
36-hr	129613	66.04	73.32	50.74	0.69	2.25	26.82	0.36	0.18	0.19
42-hr	129613	67.56	75.65	52.62	0.70	2.23	25.02	0.36	0.18	0.19
48-hr	129613	69.34	77.82	54.56	0.70	2.91	66.13	0.36	0.19	0.19
56-hr	129610	71.37	80.49	56.90	0.71	4.04	158.76	0.36	0.19	0.20
64-hr	129610	73.66	82.67	58.58	0.71	3.92	144.05	0.36	0.19	0.20
72-hr	129610	75.69	84.80	60.17	0.71	3.82	132.23	0.36	0.19	0.20



Summary statistics of sliding maxima at various time scales.

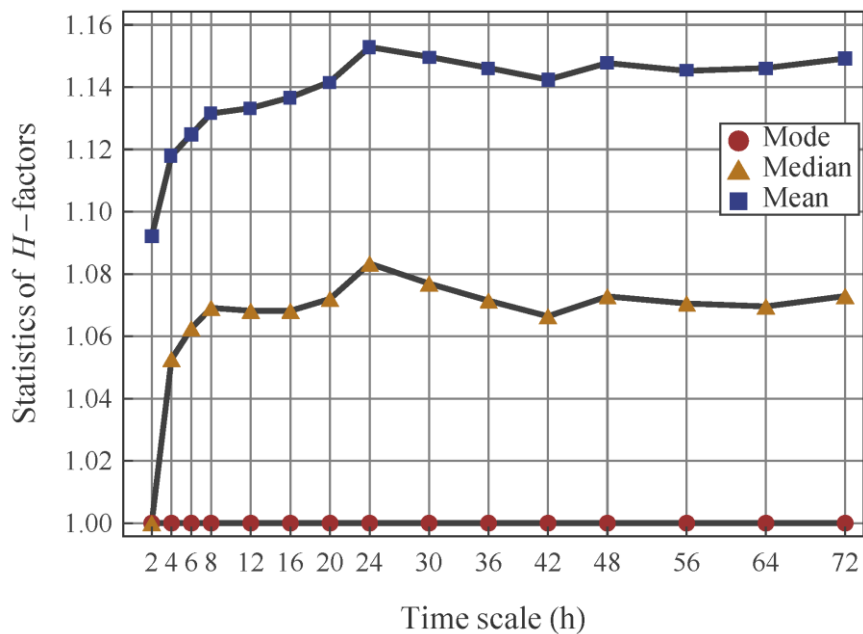


L-ratios  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  of sliding maxima at various time scales.

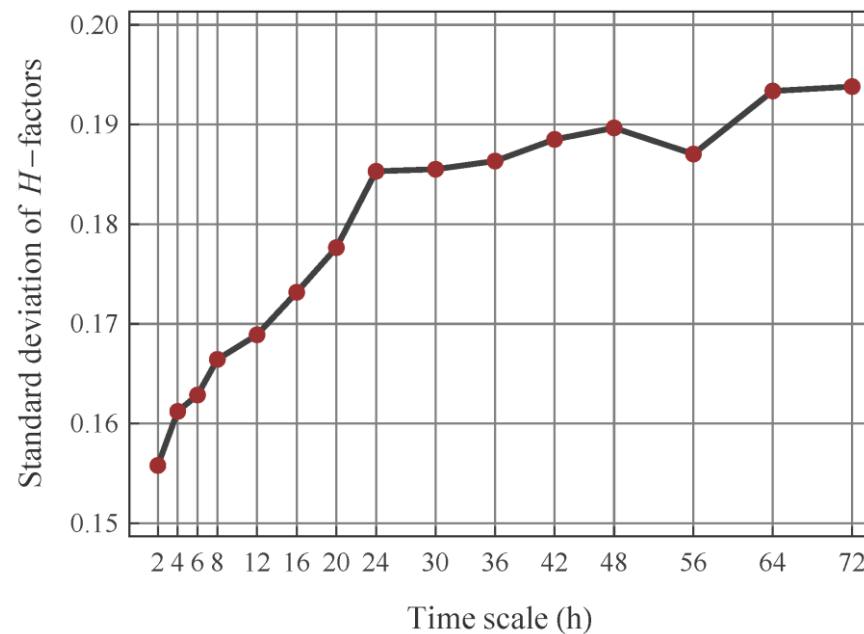
# STATISTICS OF H-FACTORS

**Table 3.** Basic summary statistics of H factors at various time scales

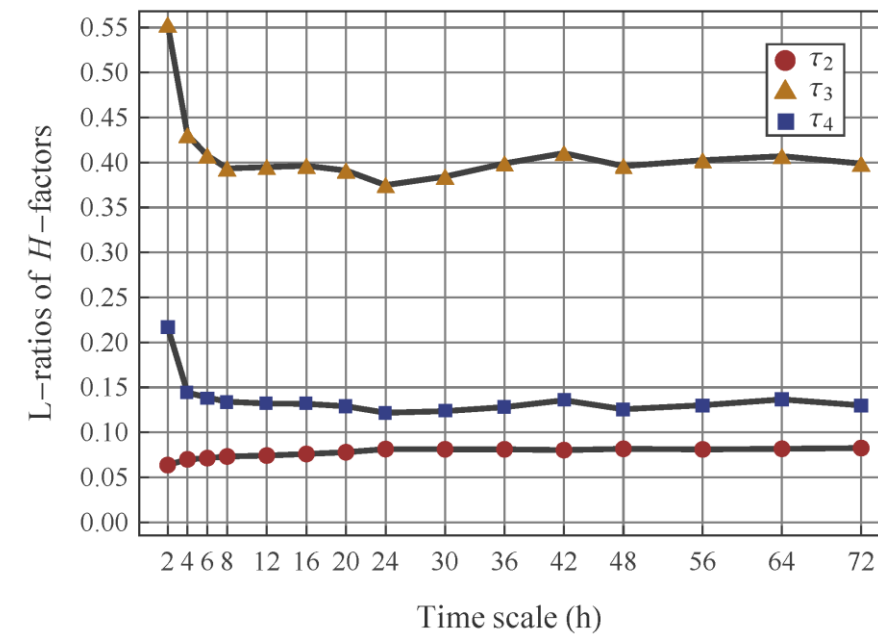
Scale	No of yrs	$H = 1$ (%)	Mode	Median	Mean	SD	$C_V$	$C_s$	$C_k$	$\tau_2$	$\tau_3$	$\tau_4$
2-hr	107684	55.7	1	1.00	1.09	0.16	0.14	2.17	7.98	0.06	0.55	0.22
4-hr	101631	36.4	1	1.05	1.12	0.16	0.14	1.89	6.89	0.07	0.43	0.14
6-hr	99806	31.3	1	1.06	1.12	0.16	0.14	1.81	6.51	0.07	0.41	0.14
8-hr	98911	28.3	1	1.07	1.13	0.17	0.15	1.75	6.23	0.07	0.39	0.13
12-hr	98753	27.9	1	1.07	1.13	0.17	0.15	1.73	6.07	0.07	0.39	0.13
16-hr	98290	27.1	1	1.07	1.14	0.17	0.15	1.70	5.86	0.08	0.40	0.13
20-hr	98139	26.3	1	1.07	1.14	0.18	0.16	1.67	5.69	0.08	0.39	0.13
24-hr	97482	23.9	1	1.08	1.15	0.19	0.16	1.59	5.33	0.08	0.37	0.12
30-hr	97927	25.9	1	1.08	1.15	0.19	0.16	1.64	5.82	0.08	0.38	0.12
36-hr	98427	27.9	1	1.07	1.15	0.19	0.16	1.69	6.13	0.08	0.40	0.13
42-hr	98718	28.9	1	1.07	1.14	0.19	0.17	3.05	61.30	0.08	0.41	0.14
48-hr	98532	28.2	1	1.07	1.15	0.19	0.17	2.98	75.69	0.08	0.40	0.13
56-hr	98578	28.7	1	1.07	1.15	0.19	0.16	1.83	8.56	0.08	0.40	0.13
64-hr	98737	15.8	1	1.07	1.15	0.19	0.17	2.65	25.95	0.08	0.41	0.14
72-hr	98319	12.9	1	1.07	1.15	0.19	0.17	2.69	36.25	0.08	0.40	0.13



Summary statistics of  $H$ -factors vs. time scale.

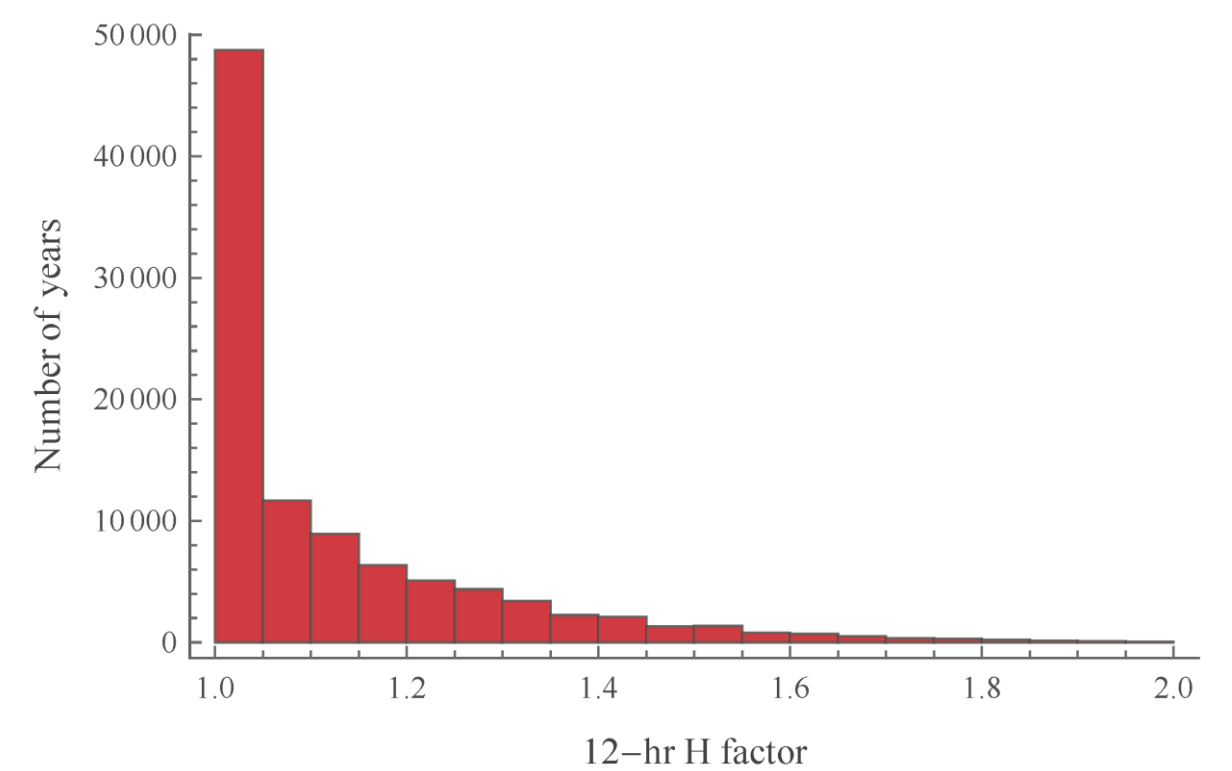
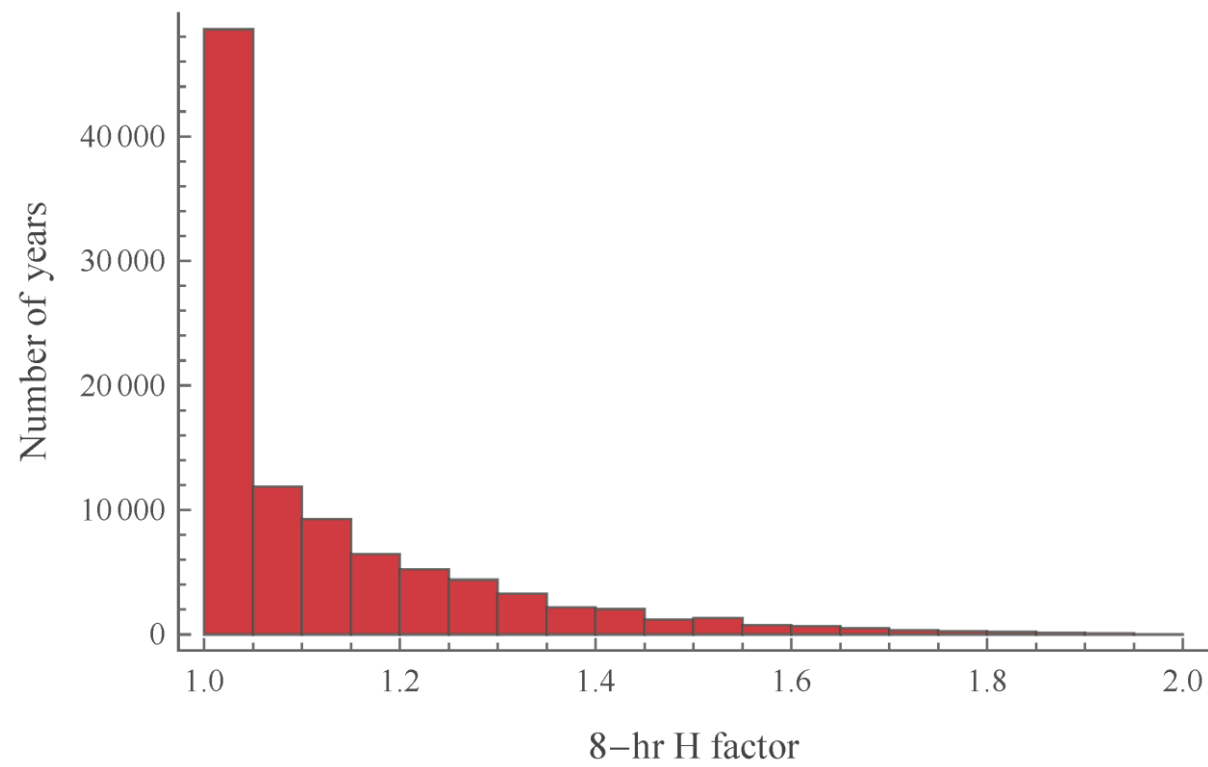
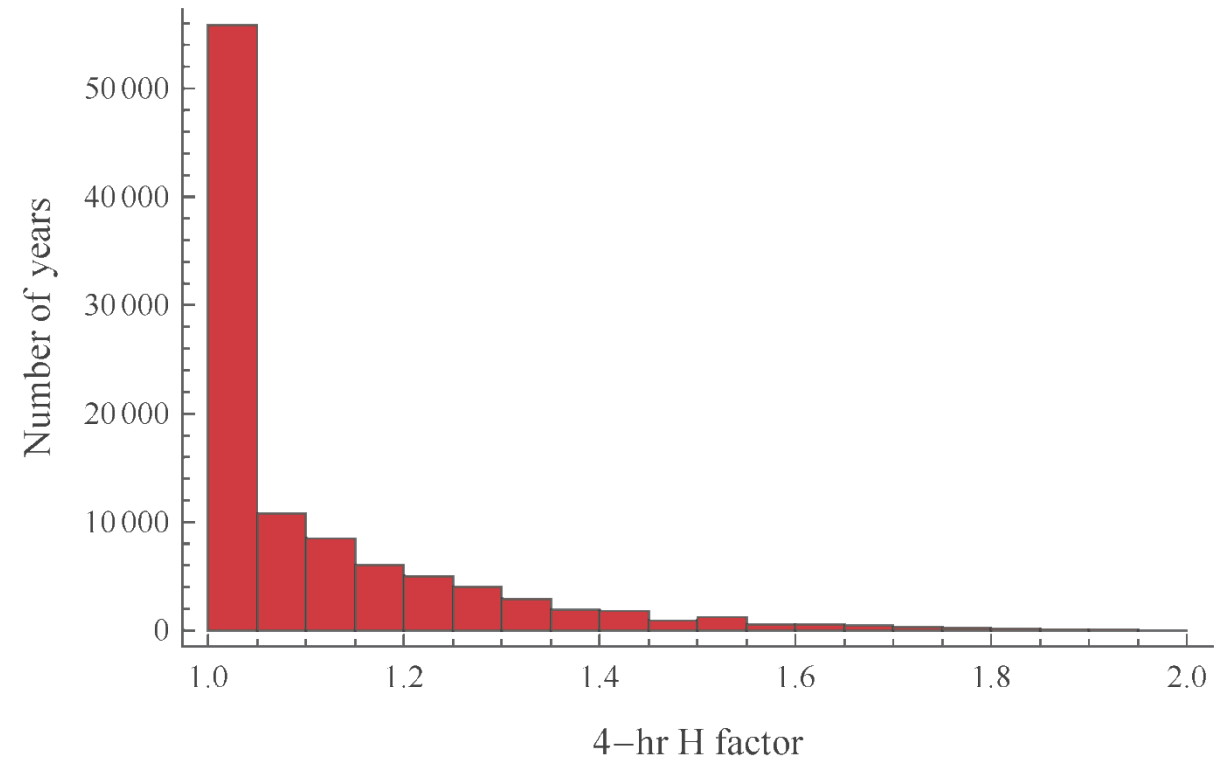
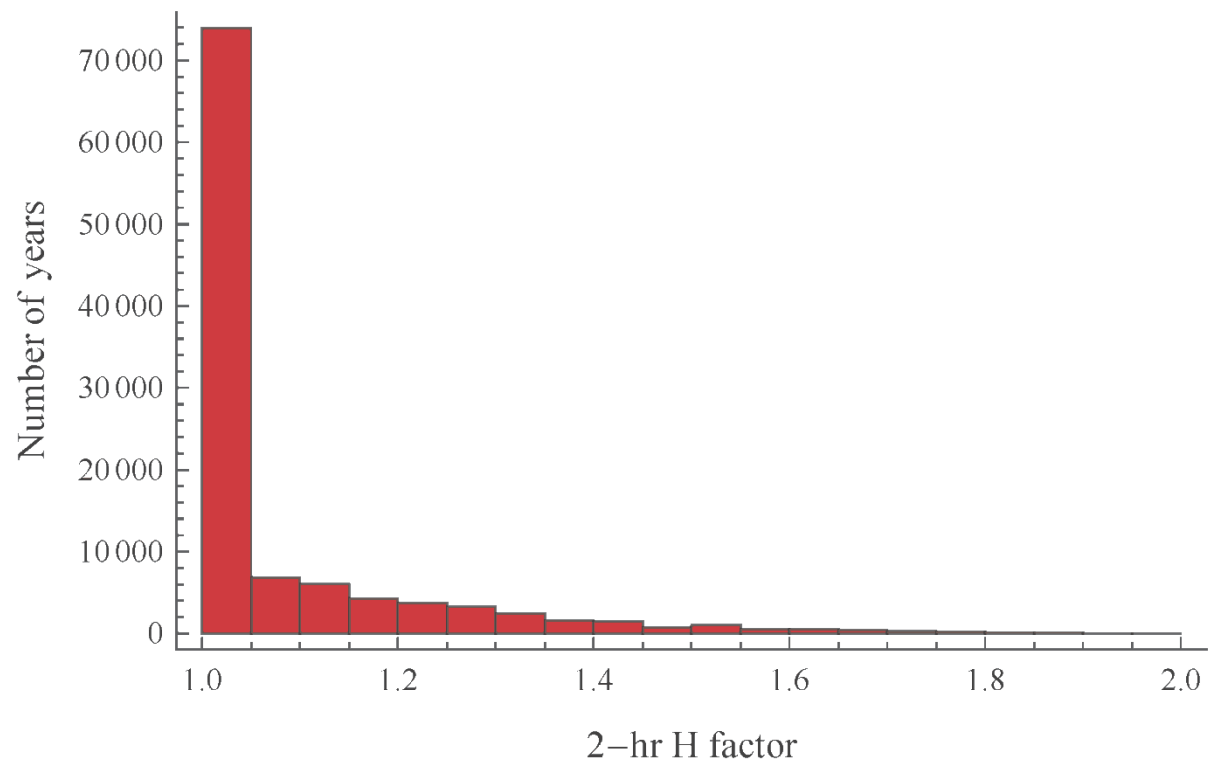


Standard deviation of  $H$ -factors vs. time scale.



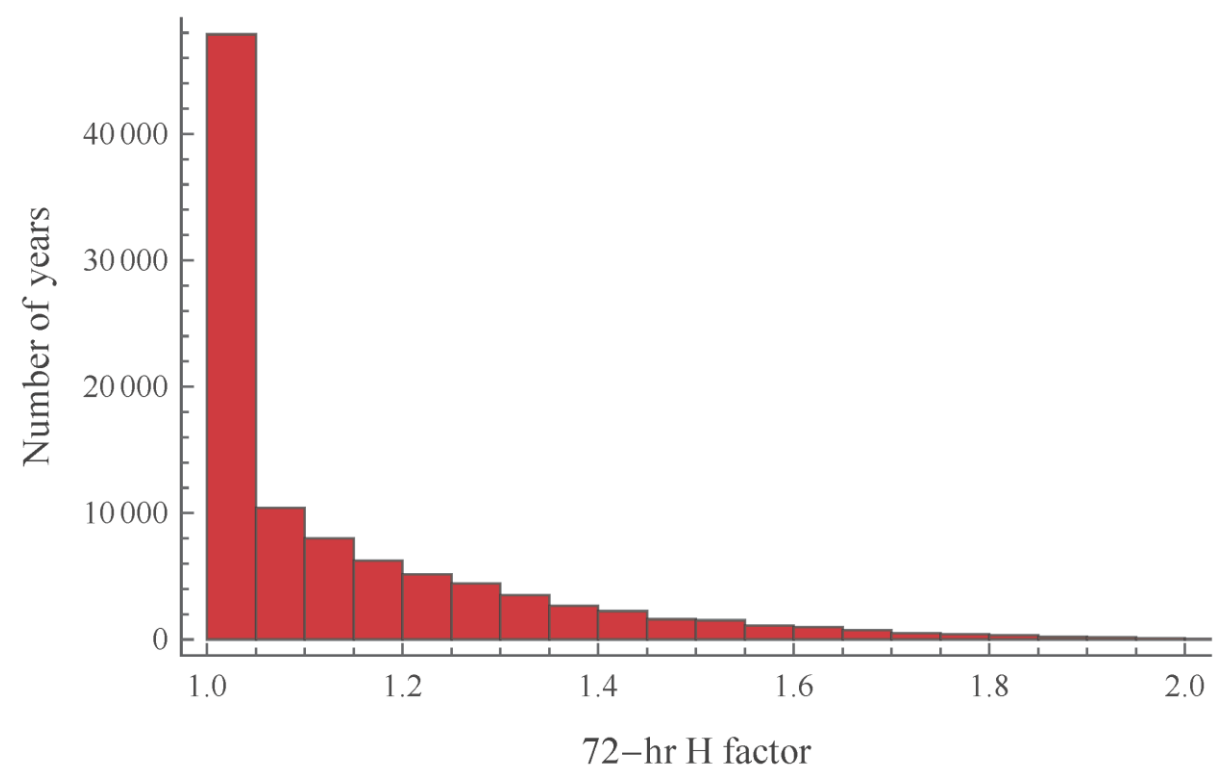
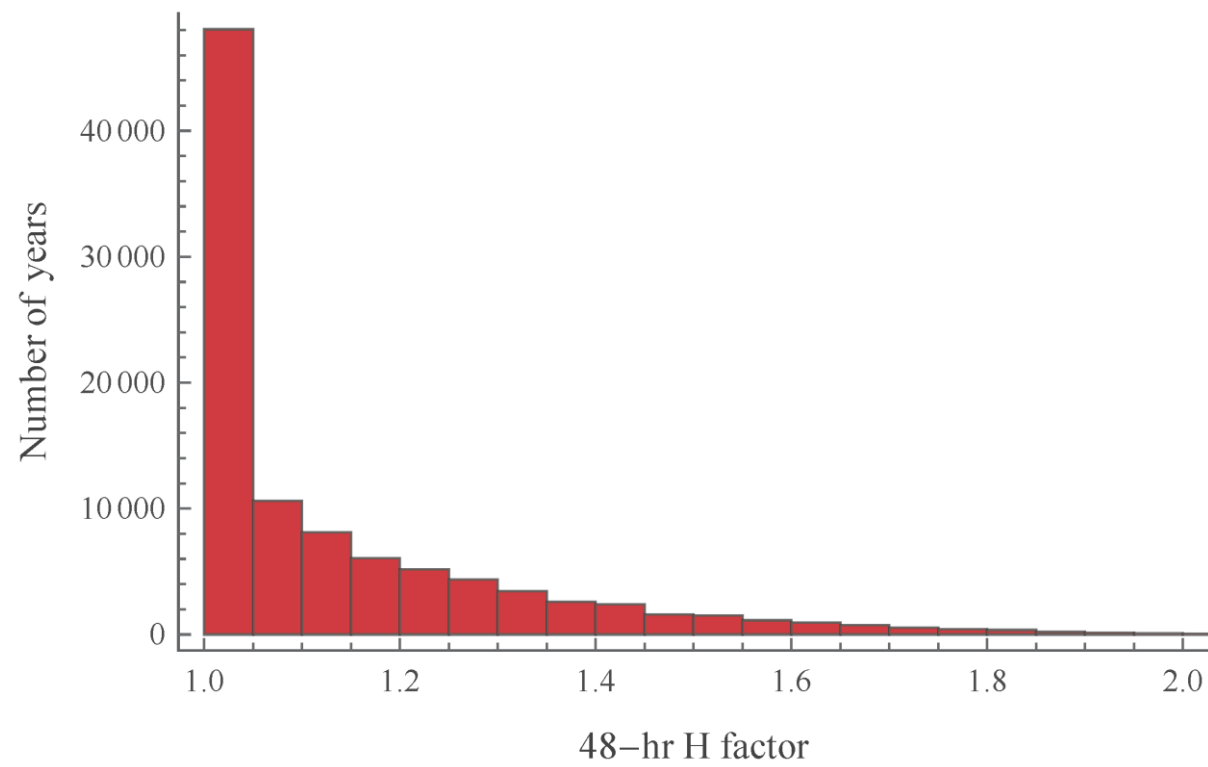
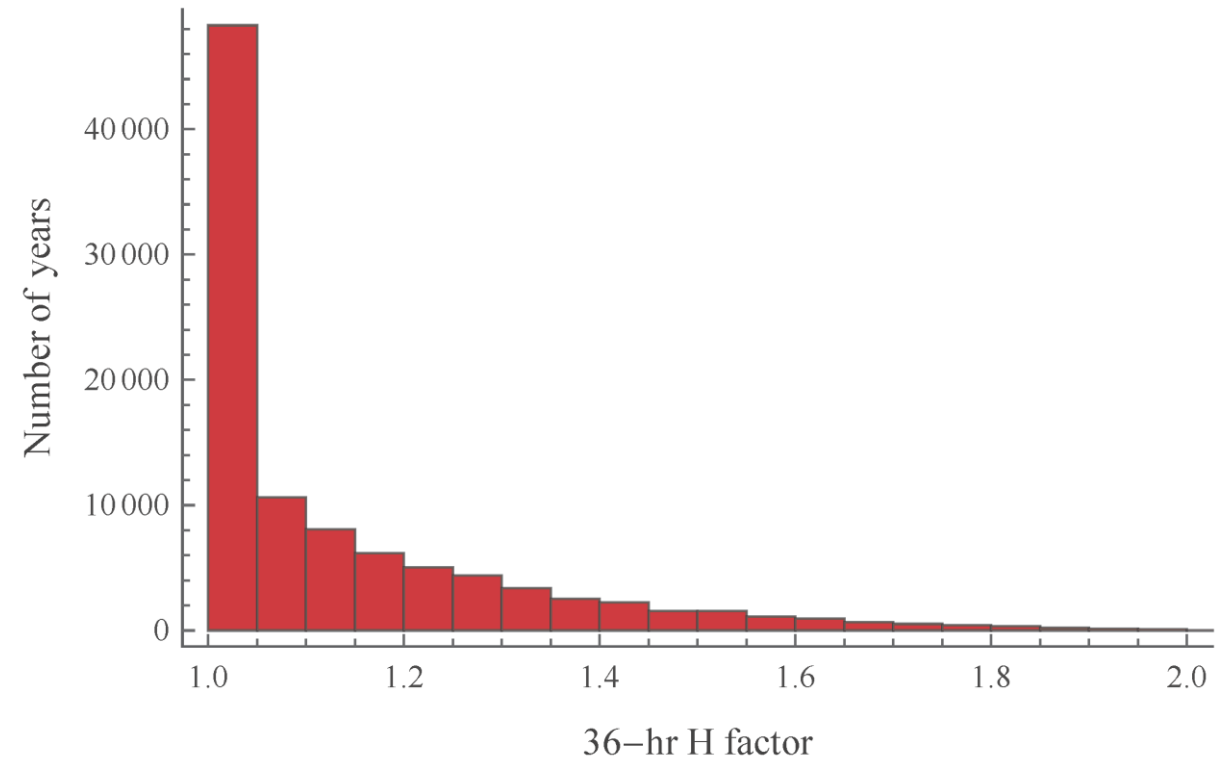
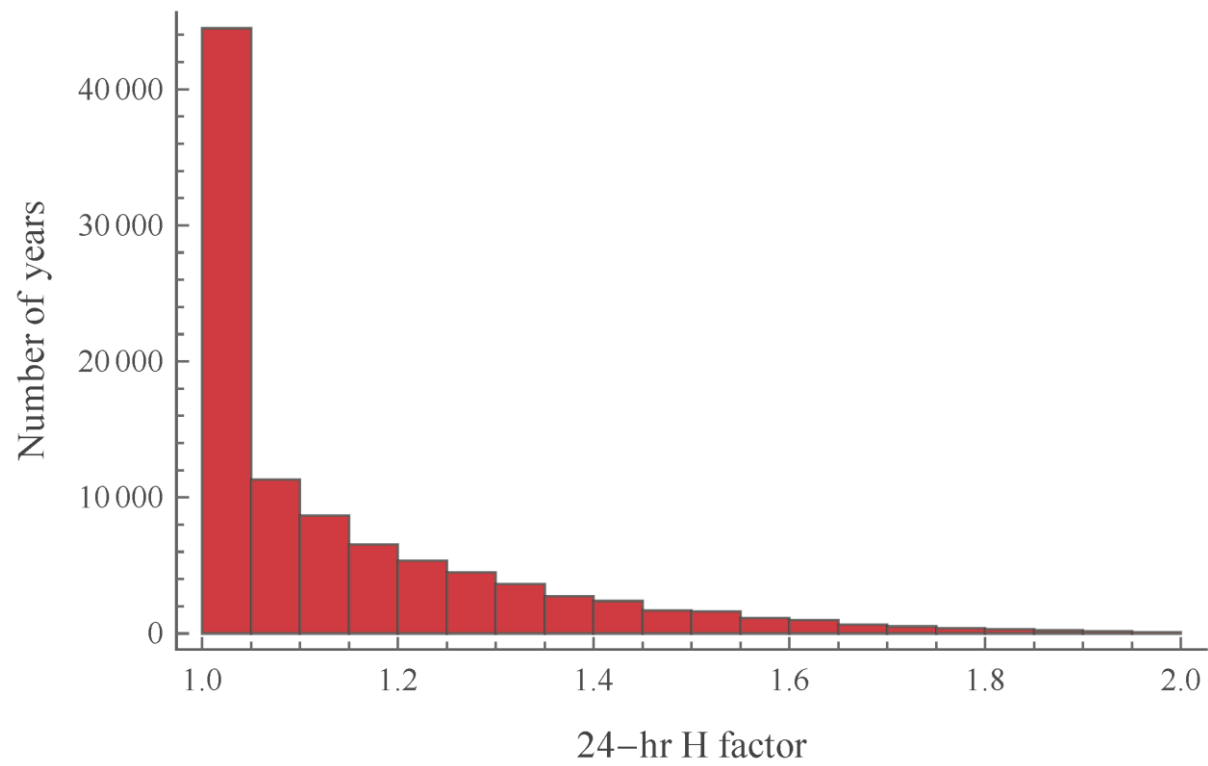
L-ratios  $\tau_2$ ,  $\tau_3$  and  $\tau_4$  of  $H$ -factors vs. time scale.

# EMPIRICAL DISTRIBUTION OF H-FACTORS (I)



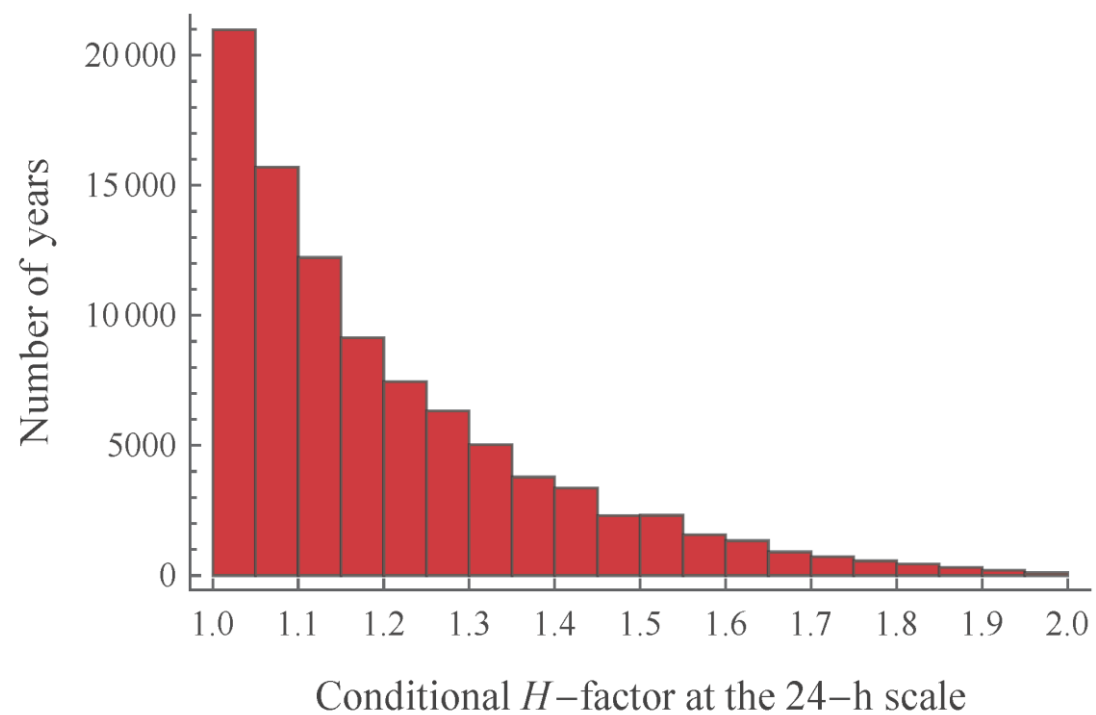
Empirical distributions of the estimated  $H$  factors for the following time scales: 2 h, 4 h, 8 h and 12 h.

# EMPIRICAL DISTRIBUTION OF H-FACTORS (II)



Empirical distributions of the estimated  $H$  factors for the following time scales: 24 h, 36 h, 48 h and 72 h.

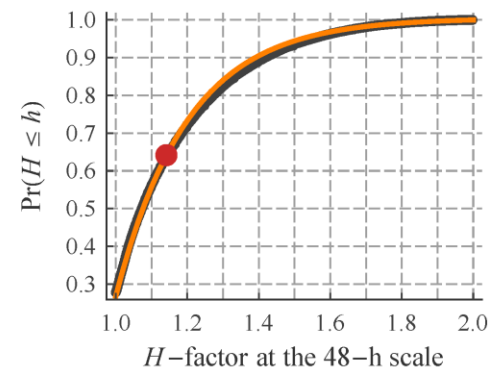
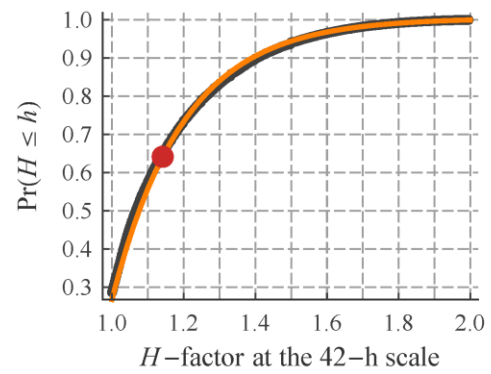
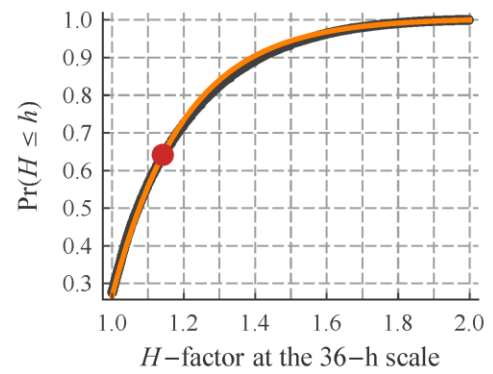
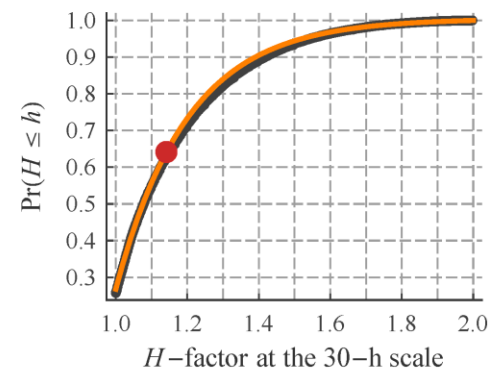
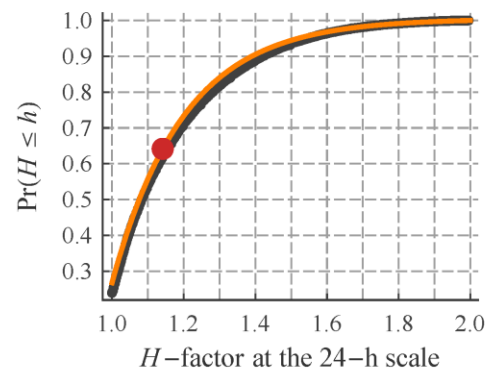
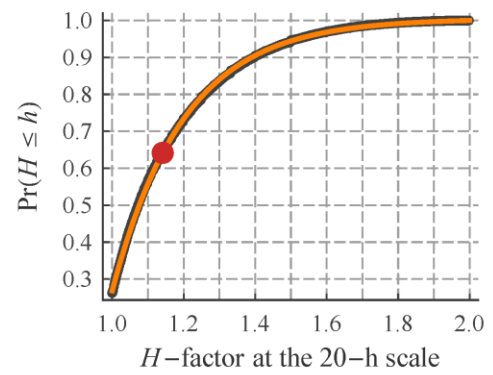
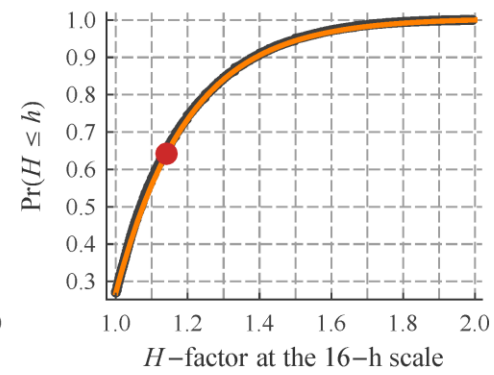
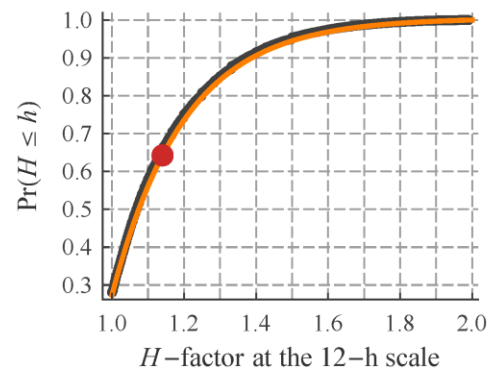
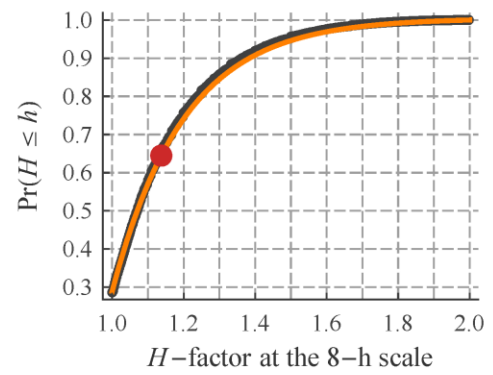
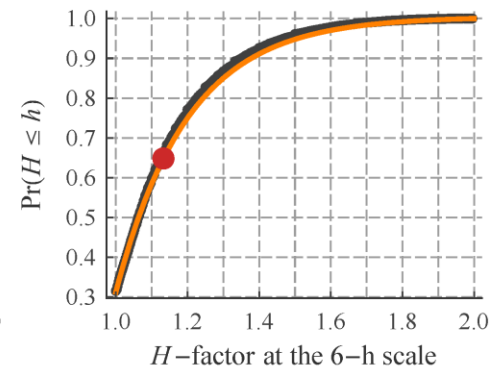
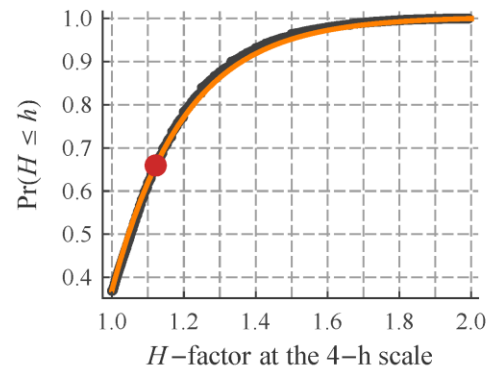
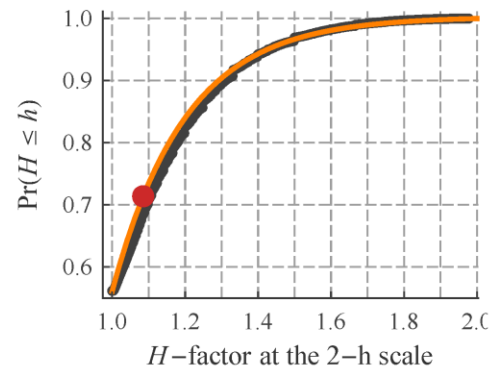
# PROBABILITY DISTRIBUTION FUNCTION OF H-FACTORS



$$F_{\text{BW}}(x) = \frac{1 - \exp\left(-\beta^{-\gamma} (x-1)^{\gamma}\right)}{1 - \exp\left(-\beta^{-\gamma}\right)}$$

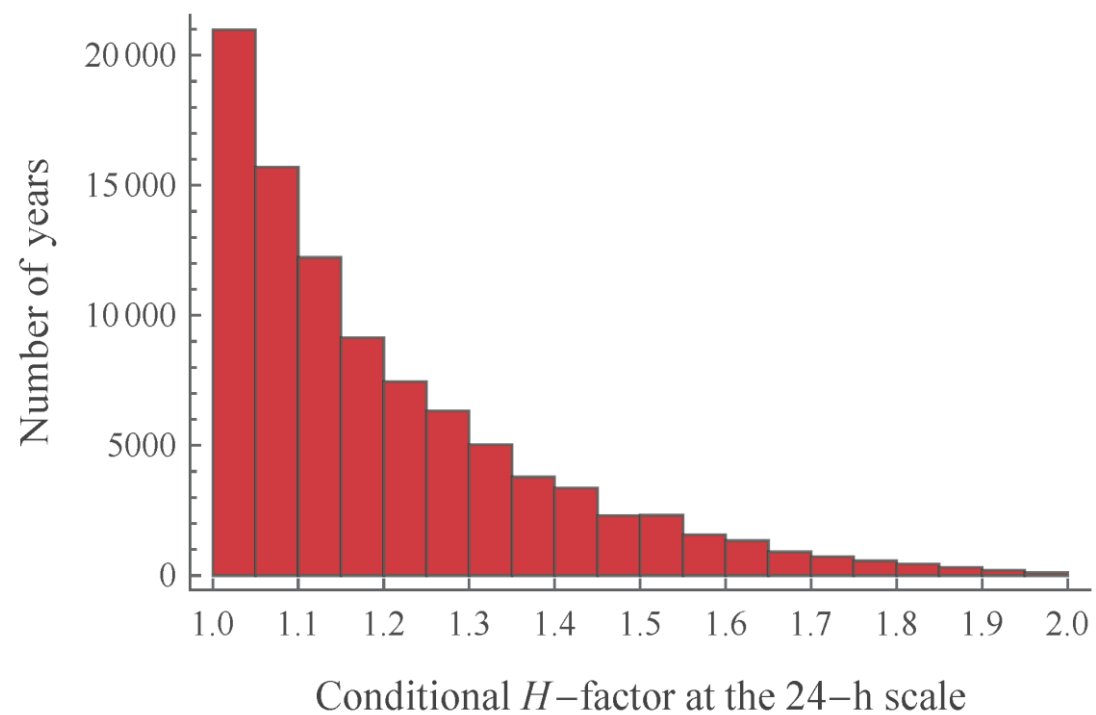
- *bounded Weibull* distribution

# PROBABILITY DISTRIBUTION FUNCTION OF H-FACTORS





# PROBABILITY DISTRIBUTION FUNCTION OF H-FACTORS



$$F_{H|H>1}(x) = \frac{1 - \exp(-5(x-1))}{1 - \exp(-5)}$$

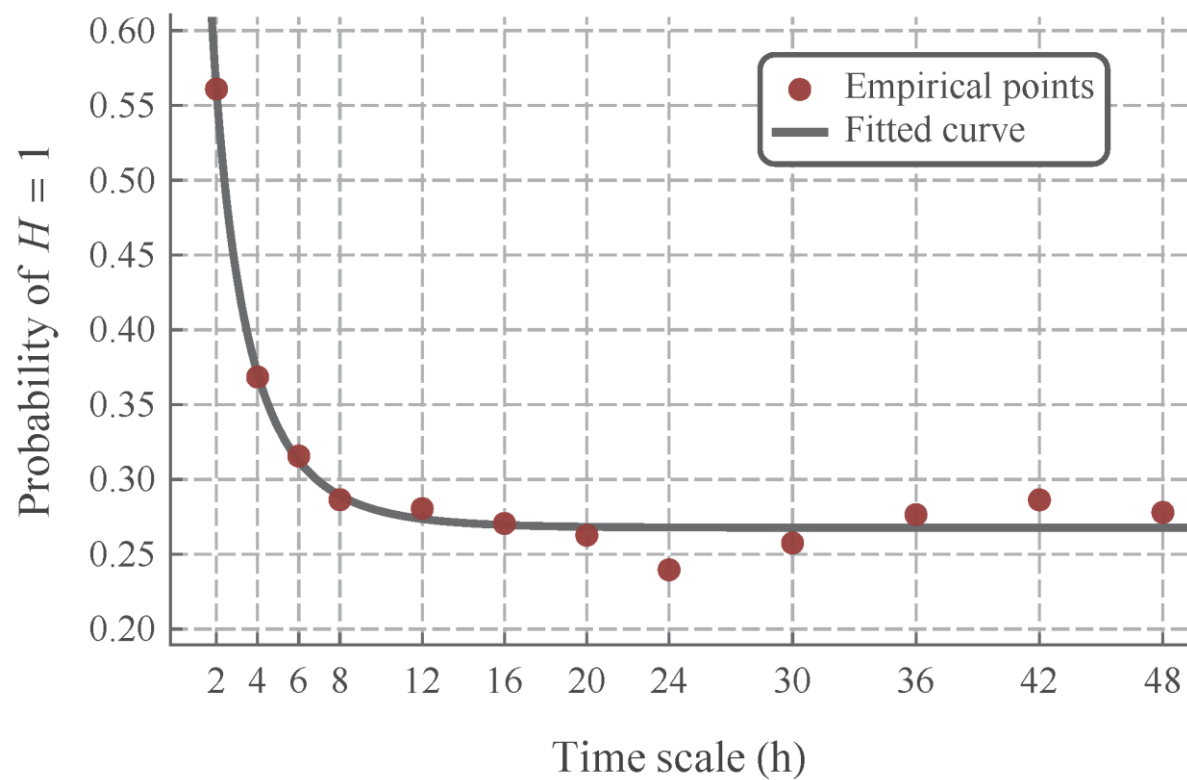
$$\mu_{H|H>1} = 1 + \beta + (1 - \exp(1/\beta))^{-1} = 1.193$$

- *bounded Weibull* distribution
- *Scale invariant* distribution fit

# PROBABILITY OF $H = 1$

$$p_1(k) = \Pr(H = 1) = a + (1 - a) \exp\left(-\left(\frac{k-1}{b}\right)^c\right)$$

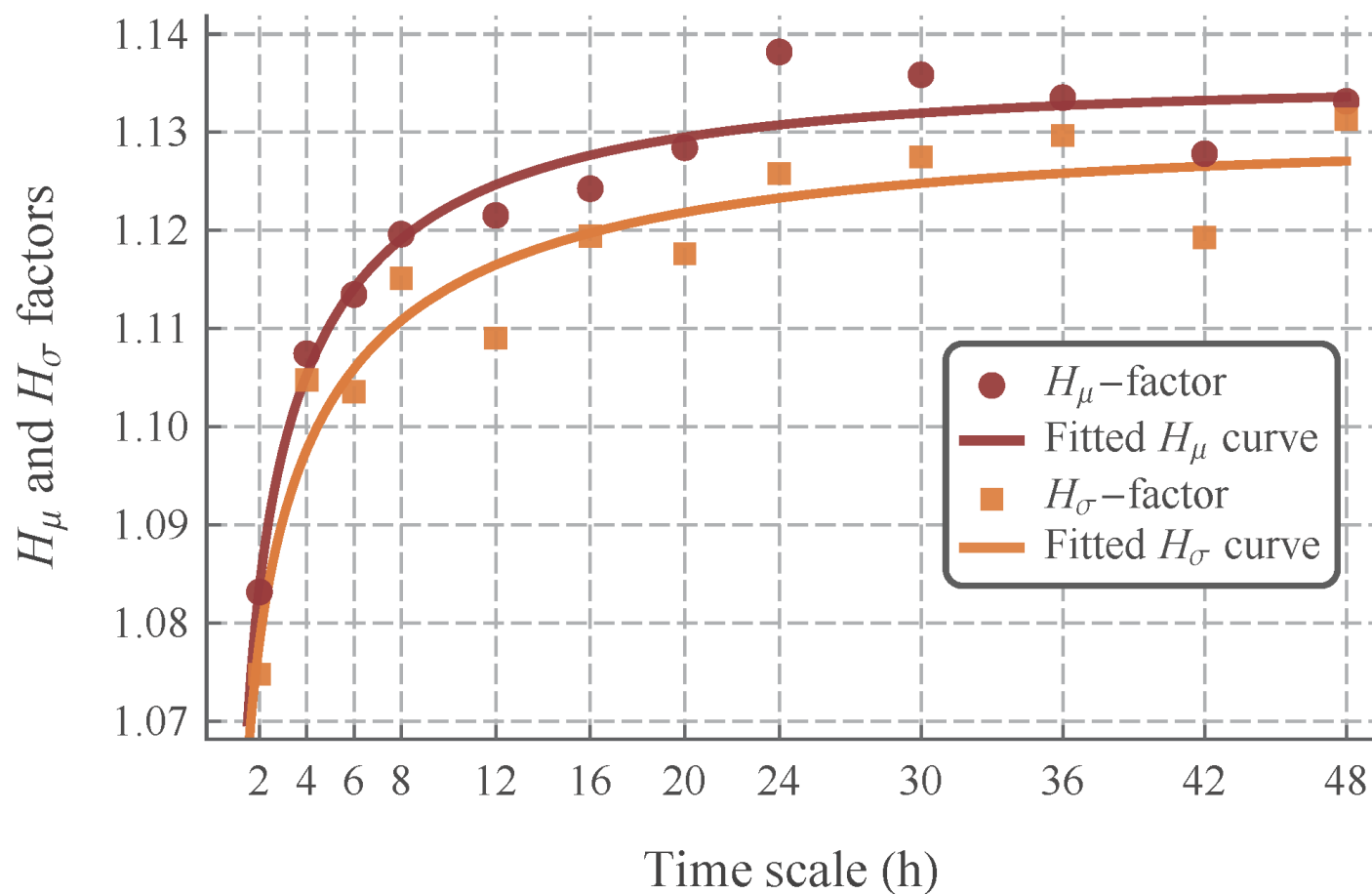
$$p_1(k) = 0.268 + 0.732 \exp\left(-\left(\frac{k-1}{1.134}\right)^{0.639}\right)$$



# COMPLETE PROBABILITY DISTRIBUTION

$$F_H(x; k) = \begin{cases} p_1(k) & H = 1 \\ (1 - p_1(k)) F_{H|H>1}(x) + p_1(k) & 1 < H \leq 2 \end{cases}$$

$$\mu_H(k) = (1 - p_1(k)) \mu_{H|H>1} + p_1(k) = 1.193 - 0.193 p_1(k)$$



# REAL WORLD EXAMPLE

- Shasta dam, California (1943-2012)
  - 100 year precipitation estimate: 268 mm → 303 mm
  - 500 year precipitation estimate: 342 mm → 386 mm
  - 1000 year precipitation estimate: 378 mm → 427 mm

# SUMMARY AND CONCLUSIONS

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- We performed an unprecedentedly large analysis of thousands of hourly precipitation records across the USA, estimating the annual Sliding and Fixed-interval maxima for several time scales
- Distribution shape characteristics of S- and F-maxima are relatively invariant
- We explored the probabilistic behavior of Hershfield's factor, which corrects for the effects of the temporal discretization and thus can be crucial for quantifying better rainfall extremes
- On average, estimates of the maximum precipitation need to be corrected by a factor of **1.13**



Papalexiou, S. M., Y. G. Dialynas, and S. Grimaldi (2016), Hershfield Factor Revisited: Correcting Annual Maximum Precipitation, *Journal of Hydrology*, 542, 884-895



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